Chapter 11 Series Part 1

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1. The first 3 terms in the expansion of $(3 - ax)^5$, in ascending powers of *x*, can be written in the form $b - 81x + cx^2$. Find the value of each of *a*, *b* and *c*.

[5]

$(3 - ax)^5$

$= 3^5 + {}^5C_1\, 3^4 \times (-ax) + {}^5C_2\, 3^3 \times (-ax)^2$

$= 81 - 405ax + 270a^2x^2$

$b - 81x + cx^2$

$b = 81$ $\quad \cancel{-}405a = \cancel{-}81$ $\quad 270a^2 = c$

$a = \frac{1}{5}$ $\quad 270 \times \frac{1}{25} = c$

$c = \frac{54}{5}$

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2. (a) Find the first 3 terms in the expansion of $(4 - \frac{x}{16})^6$ in ascending powers of x. Give each term in its simplest form. [3]

$= 4^6 + {}^6C_1 \times 4^5 \times -\frac{x}{16} + {}^6C_2 \times 4^4 \times \frac{x^2}{16^2}$

$= 4096 - 384x + 15x^2$

(b) Hence find the term independent of x in the expansion of $(4 - \frac{x}{16})^6(x - \frac{1}{x})^2$. [3]

$= (4096 - 384x + 15x^2)(x^2 - 2 + \frac{1}{x^2})$

$= -8192 + 15$

$= -8177$

3. (a) Expand $(2 - x)^5$, simplifying each coefficient. [3]

$2^5 - {}^5C_1 \times 2^4 \times x + {}^5C_2 \times 2^3 \times x^2 - {}^5C_3 \times 2^2 \times x^3 + {}^5C_4 \times 2 \times x^4 - x^5$

$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

(b) Hence solve $\frac{e^{(2-x)^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$.

[4]

$e^{\cancel{32} - \cancel{80}x + 80x^2 - 40x^3 + \cancel{10x^4} - x^5 + \cancel{80}x - \cancel{10x^4} - \cancel{32}} = e^{-x^5}$

$e^{80x^2 - 40x^3 - x^5} = e^{-x^5}$

$80x^2 - 40x^3 - \cancel{x^5} = -\cancel{x^5}$

$40x^2(2 - x) = 0$

$40x^2 = 0$ or $x = 2$

$x = 0$

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4. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

(a) Find the term independent of *x* in the binomial expansion of $(3x - \frac{1}{x})^6$.

[2]

$(3x)^6 - {}^6C_1 \times (3x)^5 \times (\frac{1}{x}) + {}^6C_2 \times (3x)^4 \times (\frac{1}{x})^2$

$- {}^6C_3 \times (3x)^3 \times (\frac{1}{x})^3$

term independent of $x = -{}^6C_3 \times 3^3$

$= -540$

(b) In the expansion of $(1 + \frac{x}{2})^n$ the coefficient of x^4 is half the coefficient of x^6.
Find the value of the positive constant *n*.

[6]

$1^n + {}^nC_1 \times \frac{x}{2} + {}^nC_2 \times \frac{x^2}{4} + {}^nC_3 \times \frac{x^3}{8} + {}^nC_4 \times \frac{x^4}{16} + {}^nC_5 \times \frac{x^5}{32} + {}^nC_6 \times \frac{x^6}{64}$

${}^nC_4 \times \frac{1}{16} = \frac{1}{2} \times {}^nC_6 \times \frac{1}{64}$

$\frac{\cancel{n!}}{(n-4)! \times 4!} \times \frac{1}{\cancel{16}\ 1} = \frac{1}{2} \times \frac{\cancel{n!}}{(n-6)! \times 6!} \times \frac{1}{\cancel{64}\ 4}$

$\frac{1}{(n-4)(n-5) \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}} = \frac{1}{2 \times 6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1} \times 4}$

$(n-4)(n-5) = 240$

$n^2 - 5n - 4n + 20 - 240 = 0$

$n^2 - 9n - 220 = 0$ | $n = 20$ or $n = -11$ (reject)

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5. Find the coefficient of x^2 in the expansion of $(x - \frac{3}{x})(x + \frac{2}{x})^5$. [5]

$(x - \frac{3}{x})(x^5 + {}^5C_1 \times x^4 \times \frac{2}{x} + {}^5C_2 \times x^3 \times \frac{4}{x^2} + {}^5C_3 \times x^2 \times \frac{8}{x^3} + \dots)$

$(x \boxed{- \frac{3}{x}})(x^5 \boxed{+ 10x^3} + 40x + \frac{80}{x} + \dots)$

coe of $x^2 = 40 - 30$

$= 10$

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6. Given that the coefficient of x^2 in the expansion of $(1+x)(1-\frac{x}{2})^n$ is $\frac{25}{4}$, find the value of the positive integer n. [5]

$$\left(1-\frac{x}{2}\right)^n = 1^n - {}^nC_1\frac{x}{2} + {}^nC_2\frac{x^2}{4} - {}^nC_3\frac{x^3}{8} + \dots$$

$(1+x)$

$$\text{coefficient of } x^2 = {}^nC_2 \times \frac{1}{4} - {}^nC_1 \times \frac{1}{2}$$

$$\frac{25}{4} = \frac{n!}{(n-2)! \times 2! \times 4} - \frac{n!}{(n-1)! \times 2}$$

$$\frac{25}{4} = \frac{n(n-1)}{8} - \frac{n}{2}$$

$(\times 8)$

$$50 = n^2 - n - 4n$$

$$0 = n^2 - 5n - 50$$

$n = 10$ or $n = -5$ (reject)

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7. The first three terms in the expansion of $(a + bx)^5(1 + x)$ are $32 - 208x + cx^2$. Find the value of each of the integers a, b and c. [7]

$(1+x)\left(a^5 + {}^5C_1 \times a^4 \times bx + {}^5C_2 \times a^3 \times b^2x^2\right)$

$(a^5 + 5a^4bx + 10a^3b^2x^2)(1+x)$

$a^5 + a^5x + 5a^4bx + 5a^4bx^2 + 10a^3b^2x^2 + \ldots = 32 - 208x + cx^2$

$32 = a^5$

$2 = a$

$a^5 + 5a^4b = -208$

$32 + 5 \times 16 \times b = -208$

$b = -3$

$5a^4b + 10a^3b^2 = c$

$-240 + 10 \times 8 \times 9 = c$

$c = 480$